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A Practical Frame-Work for the Performance Evaluation of Classical Frequency Planning Schemes in OFDM Based on Markov's Model

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ABSTRACT
In this paper, we propose a frame-work for the performance evaluation of frequency allocation schemes in 3G LTE OFDMA systems. We first develop an analytical model for collisions in an OFDMA system for an arbitrary number of users in the different cells. We then calculate the capacity of the system using a Markov model and taking into account the inter-cell interference and its impact on the adaptive modulation. We finally apply this model to compare three frequency allocation schemes, namely reuse 1, reuse 3, and a mix of reuse 1 and 3. Our results show that a mix of reuse 1 and 3 schemes outperforms a reuse 1 scheme in terms of better cell-edge performance, and outperforms also a reuse 3 scheme by achieving an higher cell throughput.

***Keywords* –** Adaptive modulation, Frequency planning, OFDMA, Performance evaluation, Throughput.

I. INTRODUCTION

3G Long-Term Evolution (LTE) standardization effort started in late 2004 in 3GPP. The objective of this evolution is to achieve high data rates with low latency and packet optimized radio access technology. It has been agreed that Orthogonal Frequency Division Multiple Access (OFDMA) will be adopted as access technology in the Evolved Universal Telecommunication Radio Access (EUTRA) [4]. In this context, inter-cell interference is a major concern, especially for users at cell edges. A mix of frequency-reuse 1 and 3 schemes has then been proposed to avoid interference at cell edges. This consists into dividing the frequency band into two sub-bands: a frequency-reuse 1 sub-band, allocated to users at cell centre, and a frequency-reuse 3 sub-band, allocated to cell edge users [2][3]. This indeed decreases interference, but also reduces peak data rates as the frequency band is not fully used at each cell. The proposed implementation of this scheme is to assign to a user a frequency sub channel that depends on its position (path loss). In this paper, we analyze and compare three different frequency allocation schemes: reuse 1, reuse 3, and a mix of reuse 1 and 3. We begin by calculating analytically the mean number of collisions for an arbitrary number of users in each cell. We then consider a system carrying elastic (FTP-like) traffic and calculate the steady-state probabilities of the number of calls in each cell. If we take into account that the modulation is chosen depending on the Signal to Interference plus Noise Ratio (SINR), the departure rate of calls, and thus the steady-state distribution, will depend on the amount of interference. We then propose an iterative algorithm to calculate the steady-state distribution and the throughput (overall and cell-edge). Our numerical results show that reuse 1 scheme achieves higher cell throughput, however it suffers from very low cell-edge performance. On the other hand, a reuse 3 scheme decreases severely the overall throughput. We then find that a good compromise between overall throughput and cell-edge performance is found by using reuse 1 at cell center and reuse 3 at cell edges.

The remainder of this paper is organized as follows. In Section II, we calculate analytically the number of collisions knowing the number of users in each interfering cell. In section III, we evaluate the performance of the classical frequency allocation schemes, namely the reuse 1 and reuse 3 schemes, taking into account the adaptive modulation and the other-cell interference. In section IV, we present the proposed hybrid frequency scheme and evaluate its performance. Our numerical results in Section V compare the different frequency allocation schemes. Section VI eventually concludes the paper.

II. ANALYTICAL CALCULATION OF COLLISIONS

In 3G LTE systems, frequency is allocated on the basis of so-called chunks, each consisting of several adjacent subcarriers. In this section, we calculate the expected number of collisions within a frequency band, knowing that we have n interfering cells, numbered from 1 to n, the target cell being numbered 0. The frequency band of N subcarriers is then partitioned into C chunks, each containing N/C subcarriers. This allocation is made in each cell site in a centralized way so that no collision is possible between users of the same cell. However, collisions are possible if nearby cells use the same frequency band. The vector K = (K0, ..,Kn) represents the numbers of allocated chunks in the n cells.

Lemma 1: In cell 0, the expected number of chunks with collisions is:

$$E[\mathcal{C}|\mathbf{K}] = K_0\left(\sum_{i=1}^{n-1}\frac{K_i}{C} - \sum_{i\neq j}\frac{K_iK_j}{C^2} + \ldots + (-1)^{n-1}\frac{\prod_{i=1}^{n}K_i}{C^n}\right) \quad (1)$$

Proof: Let us first consider the case of one interfering cell. We now calculate the probability of having c collisions. This is equivalent to the two cells choosing independently K0 and K1 chunks from C available ones. This corresponds to a hyper-geometric distribution, where we choose K0 chunks from a population of C ones, with K1"marked" ones (already chosen by cell 1). The expected number of collisions is then equal to: E [C|K] = K0K1/C. Based on this, the proportion of common subcarriers in a given group between the two cells is equal to: K0K1/C2. Let us consider now the case of two interfering cells, numbered 1 and 2. The proportion of chunks that are common between cells 1 and 2 is similarly equal to K1K2/C2. The number of collisions in cell 0 with both cells 1 and 2 has then a hyper-geometric distribution with proportion of "marked" chunks equal to K1K2/C2, which gives a mean number of common chunks between the three cells equal to K0K1K2/C2.

III. CLASSICAL FREQUENCY ALLOCATION SCHEMES

The simplest scheme to allocate frequencies in a cellular network is to use a reuse factor of 1, i.e. to allocate all chunks to each cell, leading thus to high peak data rates. However, in case of a frequency reuse of 1, high inter-cell interference is observed, especially at the cell edge. The classical interference avoidance schemes is by dividing the frequency band into 3 equal sub bands and allocate the sub bands to cells so that adjacent cells always use different frequencies. This is called reuse 3 scheme and leads to low interference, with a price of a large loss of frequency resources. In this section, we will focus on cell 0 and evaluate the performance of these two classical frequency allocation schemes. We will consider one class of data calls (full queue FTP-like calls). We will suppose that calls arrive to the cell according to a Poisson process of intensity λ, are allocated each one chunk, and stay in the system until downloading a file of an exponentially distributed size of mean Z. If, upon a call arrival, no chunks are available, the call is blocked.

A. Reuse 1 scheme

The state of the system is described by the number U of users in the cell. The state space is then:

$$S = \{U: U \leq C\}$$

We aim to calculate the steady-state probabilities π(U).

1) Mean throughput: To evaluate the performance of the system, we first characterize the throughput of calls. Let $\bar{D}$ be the mean throughput of a call. This mean throughput depends, in addition to the offered bandwidth by subcarrier *W*, on the efficiency of the used modulation and the Bloc Error Rate (BLER). This relationship is given by:

$$\bar{D} = MWE\,[e\,(1 - BLER)] \quad \textbf{(2)}$$

Where *M* is the number of subcarriers by chunk *e* is the efficiency of the used modulation (e.g. *e* is equal to 1 bit/symbol for QPSK 1/2 and to 5 bits/symbol for 64 QAM 5/6). The BLER depends on the physical layer characteristics (used modulation and path loss) and on the amount of interference. It is then correlated with the efficiency. In 3G LTE systems, Adaptive Modulation and Coding (AMC) will be used. The choice of the modulation depends on the value of Signal to Interference plus Noise Ratio (SINR, also called C/I) through the perceived BLER: the most efficient modulation that achieves a BLER larger than 10^{-1} is used. For each SINR value, this leads to a couple of values (*e*, *BLER*), determined by link level curves *e*(*C/I*) and *BLER*(*C/I*), available in the literature [1]. This gives: *e* (1 − *BLER*) = *B*(*C/I*). When calculating the SINR, we must take into account the geometric disposition of the interfering cells and the propagation conditions. These latter are the distance between the transmitter and the receiver, the shadowing and the frequency selective fading. However, as in OFDMA the data is multiplexed over a large number of subcarriers that are spaced apart at separate frequencies, the channel consists of a set of parallel, flat and non-frequency selective fading, channels [7]. The received signal is then only impacted by slow fading. In the downlink, a base station emits, for each chunk, a Constant power *P*. The SINR in cell 0 is then equal to:

$$\frac{C}{I} = \frac{P/q_0}{\sum_i (P/q_i) + N_0} \quad (3)$$

Where N0 is the background noise and q_i is the path loss between interfering base station i and the corresponding receiver. $q_i = r_i^{\alpha} 10^{\frac{\xi_i}{10}}$, with r_i the distance from base station i to the receiver, ξ_i a normal random variable due to shadowing, with zero mean and variance ς2, and α ϵ [2, 4] a constant depending on the propagation environment. In practical cases, the impact of collisions, if present, is preponderant over the background noise ($\frac{P}{r_i^\alpha} \gg$ N0). Moreover, we can approximate the sum of log-normal variables by another log-normal variable, using the Fenton-Wilkinson method [6]. Due to the lack of space, we will not detail the proof of the following lemma.

Lemma 2: The mean throughput is calculated using:

$$\bar{D} = \sum_X D(X)\Pr(X) \quad (4)$$

Where X is a vector or zeros and ones, whose dimension is equal to the number of interfering cells and whose elements correspond each to an interfering cell. The value 1 signifies that collision occurs with the corresponding cell. *D*(X) is the throughput given the vector of collisions X:

$$D(\mathbf{X}) \simeq M.W.E_{r_0}[\int_{-\infty}^{\infty} B(10^x) f_{\Lambda(r_0)}(x)dx] \quad (5)$$

Where fΛ(.) is the pdf of the Gaussian variable Λ = Λown − Λother. Λown is N (-α log (r0), ς/10) refers to the power of the "useful" signal, and Λother to the noise plus interference, calculated using the Fenton-Wilkinson method. The expectations are obtained for any geometric setting by integrating over the surface of cell 0.

2) Probabilities of collisions: We will show in this section how to calculate the probabilities of collisions *Pr*(**X**), knowing the steady-state probabilities. Let us first begin by calculating the average load in the cell, it is equal to:

$$x_i = \frac{1}{C} \sum_{K_i \le C} K_i \, \pi(K_i) \quad (6)$$

If we consider a homogeneous network with the same load in all cells, the probability *Pr*(*k*) of having exactly *k* collisions in a given chunk can be calculated using the binomial law:

$$\Pr(k) = \binom{n}{k} x^k (1-x)^{n-k}, 0 < k \le n \quad (7)$$

Using these probabilities, we can easily calculate *Pr*(X). For illustration, if we consider a traditional hexagonal cellular network, with n_1 interfering cells in the first interfering ring, and n_2 in the second interfering ring, the probability of having exactly k_i collisions with cells of ring *i* is calculated by:

$$Pr(k_1, k_2) = \frac{\binom{n_1}{k_1}\binom{n_2}{k_2}}{\binom{n_1+n_2}{k_1+k_2}} Pr(k_1 + k_2)$$

3) Iterative resolution of the system: Having obtained the mean throughput of calls, the mean holding time of calls can then be calculated by 1/μ = Z/$\bar{D}$. We are able to calculate the steady-state probabilities π(U). In fact, as the holding time of a call does not depend on the number of users in the cell (it depends only of the loads in the interfering cells), the insensitivity property can be applied [5], leading to a product form solution. The probability of having U calls in the system is then:

$$\pi(U) = \frac{1}{G} \frac{(\lambda/\mu)^U}{U!} \quad (8)$$

Where *G* is the normalizing constant given by:

$$G = \sum_{U \in S} \frac{(\lambda/\mu)^U}{U!}$$

However, in Eqn. (8), the steady-state probabilities in the target cell are calculated using the throughput of calls (Eqn. (2)), itself calculated using the steady state probabilities in the interfering cells (Eqns. (4) and (7)). We then propose to calculate the steady-state probabilities of a homogeneous network by the following iterative algorithm:

1) Set the initial values for the iterations, e.g. with taking
   a load equal to 0.5.
2) Calculate *π* (*U*) using these initial values, and deduce the load and the throughput using Eqn. (4).
3) Inject the new value of the mean holding time into Eqn. (8) and repeat the iterations until the values converge.

4) Performance measures: The obtained steady-state probabilities are the key for obtaining several performance measures such that the blocking probability:

$$b = \pi(C) \quad (9)$$

The mean time that a session spends in the system can also
Calculated using the Little's formula:

$$T = \frac{E[U]}{\lambda(1-b)} \quad (10)$$

$E[U] = \sum_{U \in S} \pi(U)$ is the average number of calls in a cell.

B. Reuse 3 scheme

In the reuse 3 scheme, only a sub-band of *C*/3 chunks is allocated in each cell. However, the same analysis as in reuse 1 scheme holds, if we replace *C* by *C*/3 and take into account that interference is only possible from cells in the second interfering ring.

IV. FRACTIONAL FREQUENCY ALLOCATION

As stated above, a frequency reuse 1 scheme suffers from high cell-edge interference, while a reuse 3 allocation limits each cell to only a third of the total frequency band. A proposed scheme is then to use a frequency reuse of 1 at the cell centers and a frequency reuse of 3 at the cell edges. This is illustrated in Figure 1 and called partial frequency reuse.

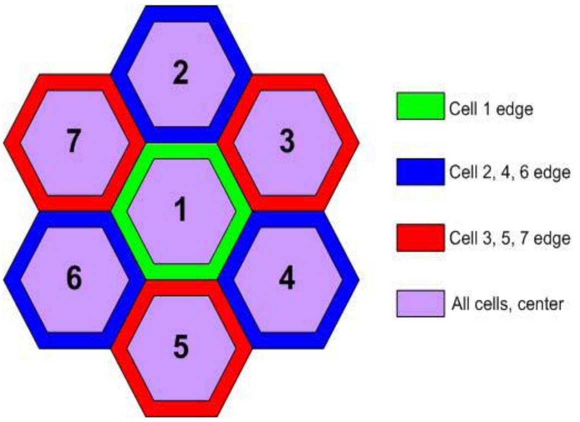

Fig. 1. Static partial frequency allocation scheme [3]

When using the above described scheme, upon the arrival of a user, it is allocated a chunk within the frequency band that corresponds to its position in the cell. As the location of the mobile cannot be precisely known, the choice is based on the path loss: A threshold on the path loss is fixed and terminal equipments with a path loss larger than this threshold are assigned a chunk within the frequency reuse 3 bandwidth. Consider now a partial frequency allocation scheme, with a frequency-reuse 1 band containing $C1$ chunks (hereafter called band 1) and a frequency-reuse 3 band containing $C2 = C-C1$ chunks (hereafter denoted band 2). A call, upon arrival, is assigned to band 1 if its path loss is less than a threshold $\hat{q}$. It is assigned to band 2 otherwise. The probability of being assigned to band 1 is then:

$$g = \Pr(q < \hat{q}) = 1 - \frac{1}{2} E_{r0}\left[erfc\left(\frac{10 \ln \frac{\hat{q}}{r_0^{\beta}}}{\sqrt{2}\varsigma \ln(10)}\right)\right] \quad (11)$$

The system is then formed by two queues corresponding to the two bands, with arrival rates $\lambda1 = g\lambda$ and $\lambda2 = (1-g)\lambda$. The same analysis as above gives the steady state probabilities and the performance measures in the two bands.

Lemma 3: The mean throughput is calculated as in Lemma 2, replacing D(X) by the new values $D^{(1)}$(X) and $D^{(2)}$(X), corresponding to the two bands:

$$D^{(k)}(\mathbf{X}) \simeq MWE_{r0}\left[\int_{\hat{q}_k^d}^{\hat{q}_k^u} \frac{f_{\Lambda_{own}}(x)}{\theta_k} \int_{-\infty}^{\infty} B(10^{x-y}) f_{\Lambda_{other}}(y) dy dx\right] \quad (12)$$

With $\hat{q}_1^u = \hat{q}_2^d = \log(\hat{q})$, $\hat{q}_1^d = -\infty$ and $\hat{q}_2^u = +\infty$. $\theta_k = \int_{\hat{q}_k^d}^{\hat{q}_k^u} f_{\Lambda_{own}}(x) dx$. The expectations are calculated over cell 0. *Proof:* The calculation must be performed taking into account that users in the two bands receive signals from different cells. Users in band 1 receive interference from neighboring cells, while those in band 2 are subject to interference from farther ones. In addition to that, when calculating the rate in each of the sub-bands, the probability distribution function must be divided by the probability of falling within the corresponding interval $[\hat{q}_k^d, \hat{q}_k^u]$, which is equal to θ_k.

V. RESULTS AND DISCUSSION

To compare the three allocation schemes, we consider a 3G LTE cellular network with a cell radius of 0.5 km. The frequency band of 10 MHZ is divided into 30 chunks of 20 subcarriers each, as specified in [4]. Each chunk has then a bandwidth of 0.3 MHz. Elastic (FTP-like) users arrive to the system with a Poisson rate and stay in it until they transmit a file of mean size of 4.5 MByte, so that it is transmitted within 2 minutes if allocated one chunk with a QPSK 1/2 modulation. Using link level simulation curves, we can choose, for each C/I value, the corresponding modulation and coding scheme and the resulting BLER, knowing that the maximal allowed BLER is of 10%. To calculate the departure rate of the users, we then divide the cell surface into a grid and integrate the resulting curve for the different values of shadowing.

A. Impact of inter-cell interference on the capacity

The other cell interference has a large impact on the capacity in OFDMA systems. To illustrate this impact, plot in Figure 2 the throughput (in bits/sec) of an isolated cell function of the arrival rate of calls, compared with its throughput when other-cell interference is taken into account. Other-cell interference decreases the throughput significantly.

B. Comparison of reuse 1, reuse 3 and reuse 1/3

First consider a homogeneous system and compare reuse 1 and static reuse 1/3 schemes. In this latter, the bandwidth is divided into four sub bands, one of 18 chunks and three of 4 chunks each. Central users are then allocated a chunk among 18 ones, while cell-edge users are allocated 4 chunks. Each cell uses then only 22 chunks. Path loss ratio is calculated so that cell edge users occupy $\frac{4}{22}$ of the cell surface.

Figure 3 shows the cell throughput for reuse 1 and reuse 1/3 schemes, function of the arrival rate of calls. Full allocation outperforms the partial one in terms of higher cell throughput is noticed. The loss in frequency resources (the 8 unused chunks) cannot then be balanced by the decrease in the interference. However, this overall throughput is not the sole determining factor in 3G LTE systems, as cell-edge performance is an important issue. Figure 4 represents the throughput of a cell-edge user. Reuse 3 scheme at cell edge increases significantly the throughput (by a factor of 20 at low loads and of 12 at high loads). A compromise is to be found with the total cell capacity loss.

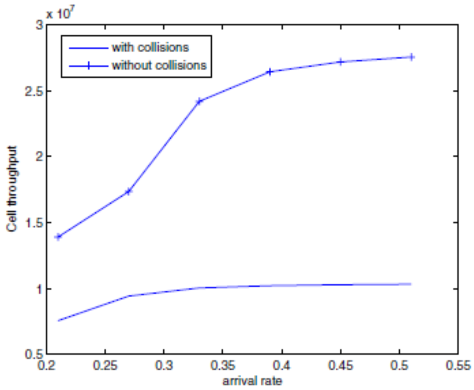

Fig.2: Impact of the other-cell interference on the cell throughput

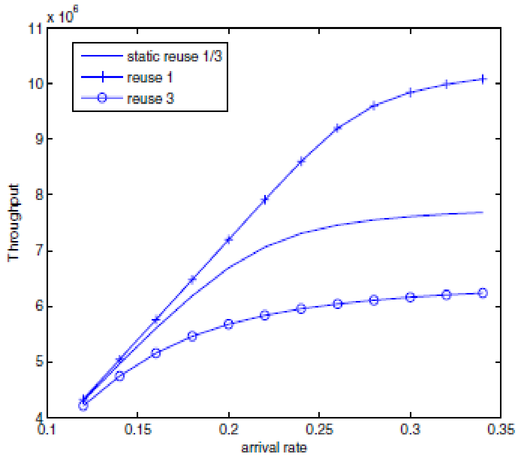

Fig. 3: Cell throughput for both static reuse 1/3 and reuse 1 schemes

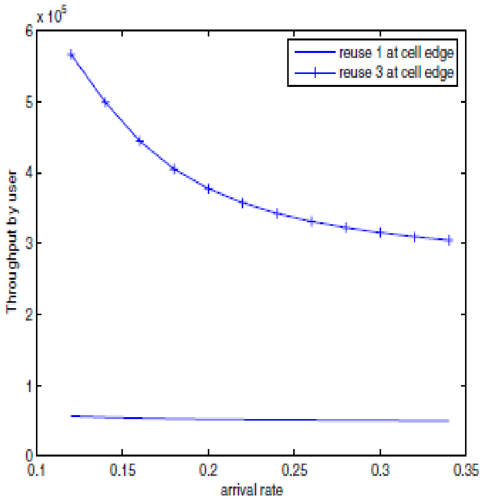

Fig. 4: Throughput of the cell edge user

VI. CONCLUSION

In this paper, different frequency allocation schemes in 3G LTE cellular systems, namely a full reuse 1 allocation, a reuse 3 allocation, and a mix of reuse 1 and 3 schemes are studied. Begin by calculating the expected number of collisions for an arbitrary number of users in the interfering cells, then considered a cellular system with elastic traffic and calculated the performance measures using a Markovian approach and taking into account the physical layer (propagation conditions and adaptive modulation). Our numerical results show that a partial frequency reuse increases substantially cell-edge performance, at the cost of lower overall capacity compared to a reuse 1 scheme. However, a reuse 1/3 scheme outperforms a classical reuse 3 scheme by achieving better cell throughput and is thus preferable.

As of future work, we aim at studying the partial frequency reuse as a scheduling scheme, where all resources are used at each cell but with a power control that reduces interference at cell edges.

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